Published by Institute of Physics Publishing for SISSA/ISAS

Received: *June 19, 2002*
Accepted: *July 23, 2002*

Intersecting S-brane solutions of $D = 11$ supergravity

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Abstract: We construct all possible orthogonally intersecting S-brane solutions in 11-dimensions corresponding to standard supersymmetric M-brane intersections. It is found that the solutions can be obtained by multiplying the brane and the transverse directions with appropriate powers of two hyperbolic functions of time. This is the S-brane analog of the "harmonic function rule". The transverse directions can be hyperbolic, flat or spherical. We also discuss some properties of these solutions.

Keywords: M-Theory, p-branes, AdS-CFT and dS-CFT Correspondence.

JHEP07(2002)038

http://jhep.sissa.it/archive/papers/jhep072002038/jhep072002038.pdf

Contents

1. Introduction

D-branes [1] have played a very important role in our understanding of non-perturbative aspects of string/M theory and recently the AdS/CFT duality [2, 3, 4]. As is well known, in perturbative string theory at weak string coupling, D-branes can be described as hypersurfaces where open strings can end. This is achieved by imposing Dirichlet boundary conditions along transverse spacelike directions in the string world-sheet action. On the other hand, D-branes have an alternative description as soliton solutions of low energy supergravity theories. These solutions are important in determining the holographic properties of D-branes. Having two different pictures for D-branes is the essence of the AdS/CFT duality.

Naturally, in string perturbation theory, one can also consider open strings obeying Dirichlet boundary conditions along time-like or null directions. These are space-like or null analogs of D-branes and are called S-branes [5] or N-branes [6], respectively. Sp-branes can be viewed as spacelike p-branes (with $p+1$ spatial dimensions) which exist only for a moment in time. In world-sheet conformal field theory, their description is very similar to usual D-branes. For instance, it is possible to describe S-branes by a boundary state. S-branes can also be considered as time dependent tachyonic kink solutions of the unstable D-brane world-volume theories [7].

S-branes are expected to play the role of D-branes in realizing dS/CFT duality [8] in string/M theory. As D-branes lead us to holography in space-like directions, one hopes that S-branes will imply time-like holography. Moreover, formulating string theory on time dependent backgrounds is important in searching cosmological applications of the theory (for some recent developments see for example [9]–[13]).

If S-branes are stable objects (even though they are not supersymmetric), then one should be able to construct corresponding supergravity solutions. In [14, 15] timelike T-dual of type II string theories were considered and euclidean brane solutions were constructed. This can be generalized to usual type II and M theories as was done in [5, 16, 17] (see also [18]–[22] for earlier work). In [5] S0-brane solution of $D = 4$ Einstein-Maxwell theory and SM5-brane solution of $D = 11$ supergravity were found. Later, all SD-brane solutions were constructed in [16] and [17]. In [16] these were obtained by solving Einstein's equations for a dilaton, gravity and an arbitrary rank antisymmetric tensor field system in an arbitrary dimension. Whereas in [17] solution generating techniques are applied to an appropriate time-dependent solution of the 11-dimensional vacuum Einstein equations. The connection between solutions of [16] and [17] is shown in [23].

Following these developments, a natural direction of study is to search for supergravity solutions of intersecting spacelike brane configurations. Intersecting solutions enriched our understanding of AdS/CFT duality and something similar is likely to happen in the context of dS/CFT correspondence too. In [17], multiply charged SDp/SD$(p-2)$ and SDp/SD$(p-4)$ brane solutions are constructed. In this paper, we will construct all possible orthogonally intersecting S-brane solutions of $D = 11$ supergravity theory corresponding to supersymmetric M-brane intersections. Our main observation is that all solutions can be obtained by a simple procedure of multiplying the brane and the transverse directions by two hyperbolic functions of time. This is the S-brane analog of the harmonic function rule [24, 25].

The organization of the paper is as follows; in the next section we give the main construction rules for intersecting S-brane solutions. In section 3, using these rules we write down the solutions explicitely and discuss their properties. We conclude in section 4. Technical details are presented in the appendix.

2. Basic rules

In this section we will consider all possible orthogonal intersections of SM2 and SM5-branes in 11-dimensions corresponding to standard supersymmetric M-brane intersections [24, 25, 26]. As we will comment, other cases turn out to have technical difficulties and cannot be obtained by the rules of this section. Similar to the usual brane intersections, one can define *common tangent*, *relative transverse* and *overall transverse* directions in an obvious way. Following the convention in the literature, an Sp brane is defined to have $p+1$-dimensional spatial world-volume. For each brane, we introduce three constants q_i, t_i and M_i, where q_i is the charge, t_i is an instant in time and M_i is a positive number. We will comment on the physical interpretations of t_i and M_i in the next section. Let $n+1$ be the dimension of the overall transverse directions including time. We take the n-dimensional transverse spatial hypersurface to be $\Sigma_{n,\sigma}$, that is the unit sphere, the unit hyperbola or flat space when $\sigma = 1$, $\sigma = -1$ and $\sigma = 0$, respectively. The 11-dimensional space can be parametrized by the coordinates $(x_1, \ldots, x_p, y_1, \ldots y_q, t, \Sigma_{n,\sigma})$, where x and y parametrize the common tangent and relative transverse directions and (t, Σ_n) is the overall transverse space. We consider a diagonal metric where the metric functions depend only on t.

For each brane we define the following function which depends on the three constants we introduced

$$H_i = \frac{q_i^2}{M_i^2} \cosh^2 \left[M_i (t - t_i) \right] . \tag{2.1}$$

In the construction, we will need one more function which characterizes the transverse space

$$G_{n,\sigma} = \begin{cases} \frac{2n(n-1)}{M^2}\sinh^2\left[M\sqrt{\frac{(n-1)}{2n}}t\right] & \sigma=-1 \text{ (hyperbola)}\,, \\ \frac{2n(n-1)}{M^2}\cosh^2\left[M\sqrt{\frac{(n-1)}{2n}}t\right] & \sigma=1 \text{ (sphere)}\,, \\ e^{2M\sqrt{\frac{(n-1)}{2n}}t} & \sigma=0 \text{ (flat)}\,, \end{cases} \tag{2.2}$$

where

$$M^2 = \sum_i M_i^2\,. \tag{2.3}$$

In terms of these functions, the metric corresponding to any number of possible standard intersection of SM-branes can be obtained by the following simple rules. Up to the H-functions, the overall transverse space takes the form

$$G_{n,\sigma}^{-\frac{n}{(n-1)}}\left[-dt^2 + G_{n,\sigma}d\Sigma_{n,\sigma}^2\right] \tag{2.4}$$

where $d\Sigma_{n,\sigma}^2$ is the metric on the unit sphere, the unit hyperbola or flat space. One can find the dependence of the metric on H-functions simply by multiplying the brane and the transverse directions for i'th brane by appropriate powers of H_i which are

$$\text{For i'th SM2-brane:}\quad \begin{cases} \text{Brane direction} & H_i^{-1/3} \\ \text{Transverse direction} & H_i^{1/6} \end{cases} \tag{2.5}$$

$$\text{For j'th SM5-brane:}\quad \begin{cases} \text{Brane direction} & H_j^{-1/6} \\ \text{Transverse direction} & H_j^{1/3} \end{cases} \tag{2.6}$$

In other words, each H function appears in the metric as $H^{1/2}$ multiplying the directions transverse to that brane and there is an overall conformal factor with $H^{-1/3}$ for SM2-brane and $H^{-1/6}$ for SM5-brane. This is very similar to the harmonic function rule of supersymmetric M-branes [24, 25]. Note that all these are valid when $n \geq 2$, and $n = 0, 1$ cases are degenerate.

After determining the metric, let us now describe the form of the anti-symmetric tensor fields. The transverse space (including time) to any SM2-brane is 8-dimensional. Let Vol_7 be the closed volume-form of the 7-dimensional spatial transverse space, where it is defined by ignoring the H and the G functions in the metric. Generically, the volume-form can be written as $\mathrm{Vol}_7 = dy_1 \wedge dy_2 \wedge \ldots \wedge dy_{7-n} \wedge \Omega_n$, where Ω_n is the volume form on $\Sigma_{n,\sigma}$. Then the four-form field of the i'th SM2-brane can be written as

$$F = q_i * \mathrm{Vol}_7\,, \tag{2.7}$$

where $*$ is the Hodge dual with respect to the full metric. Similarly, for each SM5-brane, the space-like transverse space is 4-dimensional. Defining Vol_4 to be the volume-form of this space without the H and G functions, then the four form field corresponding to the j'th SM5-brane is equal to

$$F = q_j \mathrm{Vol}_4\,. \tag{2.8}$$

It is easy to see that we have $dF = 0$ and $d * F = 0$ for both cases.

Which type of intersections are allowed for SM-branes? Since $n = 0, 1$ cases are *degenerate*, the overall transverse space (including time) should at least be 3-dimensional. Using $Sp \perp Sq(r)$ to denote intersection of Sp and Sq branes over an Sr brane, the possible double intersections are

$$\begin{aligned} &SM2 \perp SM2(0) \\ &SM2 \perp SM5(1) \\ &SM5 \perp SM5(3) \end{aligned} \tag{2.9}$$

Note that the S-brane intersection corresponding to standard $M5 \perp M5(1)$ has $n = 0$ and thus it is degenerate. For triple intersections, the only possible cases are

$$\begin{aligned} &SM2 \perp SM2 \perp SM2(0) \\ &SM2 \perp SM2 \perp SM5(0) \\ &SM2 \perp SM5 \perp SM5(1) \end{aligned} \tag{2.10}$$

The intersection of three SM5-branes over an SM3 brane has $n = 0$ and thus this configuration is degenerate. Finally, the only possible quartic intersection is

$$SM2 \perp SM2 \perp SM5 \perp SM5(0) \tag{2.11}$$

This corresponds to the special intersection of 4 M-branes, where the solution preserves 1/8 supersymmetry and not 1/16 as one would naively expect.

Since S-branes do not preserve any supersymmetry, one may try to consider other possible intersections. Let us first determine which cases are consistent with our choice of the metric and the four-form field (2.7) and (2.8). In 11-dimensions the four-form field equations are $dF = 0$ and $d{*}F \sim F \wedge F$. However, in our ansatz we have $d{*}F = 0$ and $dF = 0$. Therefore any configuration which has $F \wedge F \neq 0$ cannot be written by using S-brane rules. Another constraint comes from the fact that the Ricci tensor of the metric is diagonal. In Einstein equations, there are terms coming from the $F_{AMNP}F_B{}^{MNP}$ contraction. Any brane intersection giving non-diagonal contribution through this contraction cannot be written by using S-brane rules.

For instance, consider $SM5 \perp SM5(1)$. Although this intersection is degenerate with respect to S-brane rules, one may still insist on finding a time dependent solution. The space-time is decomposed as $(11 = 2 + 4 + 4' + 1)$ and the four-form field is equal $F = q_5 \mathrm{Vol}_{4'} + q_{5'} \mathrm{Vol}_4$ and thus $F \wedge F \neq 0$. Therefore, to be able to satisfy the four-form field equations one should consider a more general ansatz. Similarly, let us also consider the intersection of two SM5-branes on an S4-brane. Then the 11-dimensional space can be decomposed into $(t, x_1, \ldots, x_5, y_1, y_2, \Sigma_3)$, where $(x_1, \ldots, x_5, y_1)$ and $(x_1, \ldots, x_5, y_2)$ are the coordinates for the two SM5-branes, respectively. Then, the four-form field becomes $F = q_5 dy_1 \wedge \Omega_3 + q_{5'} dy_2 \wedge \Omega_3$. Although the four-form field equations are satisfied, it is easy to see that in Einstein equations there appears a non-diagonal contribution along $y_1 y_2$ directions coming from the $F_{AMNP}F_B{}^{MNP}$ contraction. This term cannot be canceled by the Ricci tensor of the metric, which is diagonal in our ansatz.

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Taking into account of these constraints, one still finds, for instance, three more double intersections which are consistent with our ansatz. These are $SM2 \perp SM2(-1)$, $SM2 \perp SM5(0)$, $SM5 \perp SM5(2)$. For these cases, we could not succeed in diagonalizing the differential equations as we did for the standard SM-brane intersections. Therefore, it seems difficult to find explicit solutions. This shows that, our S-brane rules can only be used to obtain standard S-brane intersections. Let us also recall that, even for standard cases, the overall transverse space (including time) should at least be 3-dimensional.

3. Solutions in 11-dimensions

In this section, we apply the rules given above to construct intersecting solutions. Although single SM2 and SM5-brane solutions have been obtained previously, we start with the construction of these backgrounds for completeness. Let us first consider the single SM2-brane solution. In the presence of an SM2-brane, the 11-dimensional space-time can be decomposed as $(11 = 3 + 1 + 7)$. As discussed in the previous section, we introduce 3-constants q, t_0 and M. We also have $n = 7$. Then using (2.4), (2.5) and (2.7), the solution can be written as

$$ds^2 = H^{-1/3}\left[dx_1^2 + dx_2^2 + dx_3^2\right] + H^{1/6} G_{7,\sigma}^{-7/6}\left[-dt^2 + G_{7,\sigma} d\Sigma_{7,\sigma}^2\right] \tag{3.1}$$

$$F = q * \Omega_7\,, \tag{3.2}$$

where H and G are given in (2.1) and (2.2), respectively. Note that the Hodge dual $*$ is defined with respect to the full metric (3.1).

Similarly, one can consider a single SM5-brane configuration. This time we have $(11 = 6 + 1 + 4)$ splitting of space-time and $n = 4$. From (2.4), (2.6) and (2.8), we obtain

$$ds^2 = H^{-1/6}\left[dx_1^2 + \ldots + dx_6^2\right] + H^{1/3} G_{4,\sigma}^{-4/3}\left[-dt^2 + G_{4,\sigma} d\Sigma_{4,\sigma}^2\right] \tag{3.3}$$

$$F = q\Omega_4 \tag{3.4}$$

where H and G are again given in (2.1) and (2.2), respectively.

We now consider $SM2 \perp SM2(0)$ intersection. We have $n = 5$ and the space-time is decomposed as $(11 = 1+2+2+1+5)$. The solution is specified by 6-constants (q_1, t_1, M_1) and (q_2, t_2, M_2) corresponding to the first and the second SM2-branes, respectively. Using S-brane rules of the previous section we obtain

$$\begin{aligned} ds^2 = &\,(H_1H_2)^{-1/3}\left[dx^2\right] + H_1^{-1/3}H_2^{1/6}\left[dy_1^2 + dy_2^2\right] + H_1^{1/6}H_2^{-1/3}\left[dy_3^2 + dy_4^2\right] + \\ &+ (H_1H_2)^{1/6} G_{5,\sigma}^{-5/4}\left[-dt^2 + G_{5,\sigma} d\Sigma_{5,\sigma}^2\right], \end{aligned} \tag{3.5}$$

$$F = q_1 * (dy_3 dy_4 \Omega_5) + q_2 * (dy_1 dy_2 \Omega_5)\,. \tag{3.6}$$

The first SM2-brane has the charge q_1 and world-volume coordinates (x, y_1, y_2). This brane is characterized by the function $H_1(q_1, t_1, M_1)$ given in (2.1). Similarly the second SM2-brane has the charge q_2, world-volume (x, y_3, y_4) and the function $H_2(q_2, t_2, M_2)$.

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Construction of $SM5 \perp SM5(3)$ runs along similar lines. The space-time is decomposed as $(11 = 4+2+2+1+2)$ and $n = 2$. There are again 6-constants, 3 for each SM5-brane. Using S-brane rules one finds

$$\begin{aligned} ds^2 &= (H_1H_2)^{-1/6}\left[dx_1^2+\ldots+dx_4^2\right] + H_1^{-1/6}H_2^{1/3}\left[dy_1^2+dy_2^2\right] + H_1^{1/3}H_2^{-1/6}\left[dy_3^2+dy_4^2\right] + \\ &\quad + (H_1H_2)^{1/3}G_{2,\sigma}^{-2}\left[-dt^2+G_{2,\sigma}d\Sigma_{2,\sigma}^2\right], \qquad (3.7) \\ F &= q_1(dy_3dy_4\Omega_2) + q_2(dy_1dy_2\Omega_2). \qquad (3.8) \end{aligned}$$

The first SM5-brane has the charge q_1 and oriented along $(x_1,\ldots,x_4,y_1,y_2)$ hyperplane. Similarly, the second SM5-brane has the charge q_2 and world-volume directions $(x_1,\ldots,x_4, y_3,y_4)$.

Let us now obtain $SM2 \perp SM5(1)$ which is the last possible double intersection. The space-time is decomposed as $(11 = 2+1+4+1+3)$. Let q_1 and q_2 be the charges of the SM2 and SM5-branes respectively. Then the solution can be written as

$$\begin{aligned} ds^2 &= H_1^{-1/3}H_2^{-1/6}\left[dx_1^2+dx_2^2\right] + H_1^{-1/3}H_2^{1/3}\left[dy_1^2\right] + H_1^{1/6}H_2^{-1/6}\left[dy_2^2+\ldots+dy_5^2\right] + \\ &\quad + H_1^{1/6}H_2^{1/3}G_{3,\sigma}^{-3/2}\left[-dt^2+G_{3,\sigma}d\Sigma_{3,\sigma}^2\right], \qquad (3.9) \\ F &= q_1 * (dy_2\ldots dy_5\Omega_3) + q_2(dy_1\Omega_3). \qquad (3.10) \end{aligned}$$

The SM2-brane is oriented along (x_1,x_2,y_1), and SM5-brane is oriented along $(x_1,x_2, y_2,\ldots,y_5)$ hyperplanes. Again the solution depends on 6-constants, 3 for each SM-brane.

The other intersecting solutions can easily be obtained once the brane directions are specified. For $SM2 \perp SM2 \perp SM2(0)$ intersection let (x,y_1,y_2), (x,y_3,y_4) and (x,y_5,y_6) parametrize the three SM2-brane world-volumes. So, x is the common tangent direction and the overall transverse space becomes (t,Σ_3). The corresponding metric and four-form field are

$$\begin{aligned} ds^2 &= (H_1H_2H_3)^{-\frac{1}{3}}\left[dx^2\right] + H_1^{-\frac{1}{3}}(H_2H_3)^{\frac{1}{6}}\left[dy_1^2+dy_2^2\right] + H_2^{-\frac{1}{3}}(H_1H_3)^{\frac{1}{6}}\left[dy_3^2+dy_4^2\right] + \\ &\quad + H_3^{-1/3}(H_1H_2)^{\frac{1}{6}}\left[dy_5^2+dy_6^2\right] + (H_1H_2H_3)^{\frac{1}{6}}G_{3,\sigma}^{-\frac{3}{2}}\left[-dt^2+G_{3,\sigma}d\Sigma_{3,\sigma}^2\right], \qquad (3.11) \\ F &= q_1 * (dy_3dy_4dy_5dy_6\Omega_3) + q_2 * (dy_1dy_2dy_5dy_6\Omega_3) + q_3 * (dy_1dy_2dy_3dy_4\Omega_3). \qquad (3.12) \end{aligned}$$

Note that the solution depends on 9 arbitrary constants, 3 for each SM2-brane.

Finally, let us construct the quartic intersection $SM2 \perp SM2 \perp SM5 \perp SM5(0)$. As we will see, the remaining two cases in (2.10) can be obtained from this solution. We place two SM5-branes along (x,y_1,y_2,y_3,y_4,y_5) and (x,y_1,y_2,y_3,y_6,y_7) hyperplanes. Over this intersection, we add two SM2-branes oriented along (x,y_4,y_6) and (x,y_5,y_7) directions. The corresponding solution can be written as

$$\begin{aligned} ds^2 &= (H_1H_2)^{-1/3}(H_3H_4)^{-1/6}\left[dx^2\right] + (H_1H_2)^{1/6}(H_3H_4)^{-1/6}\left[dy_1^2+dy_2^2+dy_3^2\right] + \\ &\quad + H_1^{-1/3}H_2^{1/6}H_3^{-1/6}H_4^{1/3}\left[dy_4^2\right] + H_1^{1/6}H_2^{-1/3}H_3^{-1/6}H_4^{1/3}\left[dy_5^2\right] + \\ &\quad + H_1^{-1/3}H_2^{1/6}H_3^{1/3}H_4^{-1/6}\left[dy_6^2\right] + H_1^{1/6}H_2^{-1/3}H_3^{1/3}H_4^{-1/6}\left[dy_7^2\right] + \\ &\quad + (H_1H_2)^{1/6}(H_3H_4)^{1/3}G_{2,\sigma}^{-2}\left[-dt^2+G_{2,\sigma}d\Sigma_{2,\sigma}^2\right], \qquad (3.13) \\ F &= q_1 * (dy_1dy_2dy_3dy_5dy_7\Omega_2) + q_2 * (dy_1dy_2dy_3dy_4dy_6\Omega_2) + \\ &\quad + q_3(dy_6dy_7\Omega_2) + q_4(dy_4dy_5\Omega_2). \qquad (3.14) \end{aligned}$$

This solution has 12 free parameters.

In all these intersections, it is possible to remove any number of brane configurations from the system. For i'th brane if one considers the limit

$$q_i \to 0\,, \qquad M_i \to 0\,, \qquad \frac{q_i}{M_i} = 1\,, \tag{3.15}$$

then the H-function corresponding to this brane becomes $H_i = 1$. Thus all information about this brane dissappears, giving a background with one less number of intersections. In this way, one can obtain $SM2 \perp SM2 \perp SM5(0)$ and $SM2 \perp SM5 \perp SM5(1)$ intersections from the quartic intersection above.[1] Continuing like this, the flat space limit can only be obtained when $\sigma = 0, -1$ by applying the limit (3.15) to each brane. In the $\sigma = -1$ case, the limit (3.15) gives the flat metric written in Rindler coordinates i.e $-dt^2 + t^2 d\Sigma_{n,-1}$. For $\sigma = 1$, the flat space limit is singular.

There are 3-constants of integration (q_i, t_i, M_i) for each brane. It turns out that it is possible to eliminate one of the constants M_i (for instance, by setting it's value to 1). In all cases, this can be achieved by a scaling $t \to t/M_i$ followed by redefinitions $M_j \to M_i M_j$, $j \neq i$. Then by (2.3), M should also be scaled as $M \to M_i M$ so that $M^2 = 1 + \sum_{j\neq i} M_j^2$. These should be supplemented by further scalings of x and/or y coordinates or by redefinitions of charges q_i, if necessary. For example, for single SM2-brane and SM5-brane solutions $x \to x M_i^{-1/3}$ and $x \to x M_i^{-1/6}$ scalings should also be performed, respectively. Similarly, in $SM2 \perp SM5(1)$ and $SM2 \perp SM2 \perp SM2(0)$ intersections, $x \to M_i^{-1/2}$ and $x \to x/M_i$ scalings are enough to eliminate M_i in the solutions, respectively . In $SM2 \perp SM2(0)$ intersection, in addition to $x \to M_i^{-1/2} x$ scaling, $y \to M_i^{-1/8} y$ and $q \to M_i^{1/4} q$ replacements should also be performed. In all other cases, one can find similar scalings which leave the solutions invariant and eliminate one of the constants M_i. Note however that, after eliminating M_i it is not anymore possible to remove that brane from the system. In particular the flat space limit cannot be taken when $\sigma = 0, -1$.

What is the physical interpretation of these constants? It is clear that q_i is the electric or magnetic charge of the i'th brane. Since S-branes are spacelike p-branes which exist only for an instant in time, presumably t_i can be identified with (or somehow be related to) that instant.[2] Note however that the solutions are not invariant under $t \to t + constant$. This suggests that S-branes are naturally defined on a vacuum where there is no time translation symmetry. Finally, it is tempting to claim that M_i is related to the energy of the i'th SM-brane. Note that since S-branes are not supersymmetric, their masses can in principle be independent of their charges (This is similar to having any number of electrons and protons with charge $+1$ with arbitrary mass). Therefore, in S-brane solutions, there should also be a free parameter describing the energy of the brane. One support for identification of M_i

[1] However, note that not all intersections can be obtained by this procedure since it may result some smearing along overall transverse directions. For instance, removing a brane from $SM2 \perp SM2(0)$ gives the smeared version of (3.1).

[2] When t_i's are different than each other, then it might be more appropriate to call these solutions as *overlapping* S-branes.

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with the energy comes from the observation that by a scaling $t \to t/M_i$ (followed by others as explained in the above paragraph), it is possible to set $M_i = 1$. The energy operator $E = i\partial_t$ scales as $E \to M_i E$. Thus, if the i'th S-brane has energy M_i with respect to the original time t, then after scaling it should have energy 1, which is the case. Although this argument suggests the identification, to verify that M_i corresponds to the energy of the i'th S-brane requires a precise definition of mass as a conserved quantity for these time dependent backgrounds.

As in the case of usual M-brane solutions it is possible to apply dimensional reduction to these solutions to obtain intersecting S-branes of type IIA theory [27, 25]. To be able to reduce $\sigma = \pm 1$ solutions along overall transverse directions one should first find a way of smearing spherical or hyperbolic directions. For $n = 2$, smearing out gives two flat directions which reduces to the $\sigma = 0$ case. Therefore, this procedure is meaningful only when $n \geq 3$. It is straightforward to verify that upto H factors, the overall transverse space (2.4) can be flattened by one direction[3] using

$$G_{n,\pm}^{-\frac{n}{(n-1)}} \left[-dt^2 + G_{n,\pm} d\Sigma_{n,\pm}^2\right] \to dz^2 + G_{n-1,\pm}^{-\frac{(n-1)}{(n-2)}} \left[-dt^2 + G_{n-1,\pm} d\Sigma_{n-1,\pm}^2\right]. \qquad (3.16)$$

This procedure can be repeated until $\Sigma_\pm$ becomes 2-dimensional. It is now possible to reduce the solution along z coordinate to find a solution of type IIA theory. Note that, without smearing out, one can still perform a double dimensional reduction along common tangent or relative transverse directions.

A double dimensional reduction of SM2 and SM5-branes gives spacelike NS-string (SNS1) and SD4-brane of type IIA theory, respectively. A reduction along transverse directions would give SD2-brane and SNS5-brane solutions, respectively. By applying dimensional reduction along an overall transverse direction, we obtain an $SD2 \perp SD2(0)$ solution from $SM2 \perp SM2(0)$ intersection. It is now possible to apply several T-duality transformations to obtain a list of intersecting SD-branes of type IIA and type IIB-theory. On the other hand, one can also reduce $SM2 \perp SM2(0)$ solution along the common or one of the relative transverse directions and this gives $SNS1 \perp SNS1(0)$ or $SNS1 \perp SD2(0)$ intersections, respectively. In this way one can in principle obtain all possible spacelike backgrounds corresponding to standard brane intersections.

Let us now try to determine the singularities and the asymptotics of the solutions. For finite t, all metric functions are regular and non-zero except for $G_{n,-1}$ in (2.2) which vanishes at $t = 0$. However, as $t \to 0$, $G_{n,-1} \to t$ and it is easy to see that by defining a new coordinate $u \sim t^{-1/(n-1)}$ the metric asymptotes to the flat space. More precisely, along the overall transverse directions, the metric becomes $-du^2 + u^2 d\Sigma_{n,-1}$ and $u \to \infty$. Therefore, for $\sigma = -1$, $t \in (0, \infty)$ and near the first asymptotic region as $t \to 0$, the metric becomes locally flat space in Rindler coordinates. For $\sigma = 0, 1$ solutions, $t \in (-\infty, +\infty)$.

[3]It is possible to have $g_{zz}(t) \sim e^{ct}$, where c is another constant. This adds one more free constant to the list. The constant M defined in (2.3) should also be modified appropriately.

For $\sigma = 0$, as $t \to -\infty$, the metric along overall transverse space becomes[4]

$$ds^2_{n+1} \to -e^{-\alpha t} dt^2 + e^{-\beta t} d\Sigma^2_{n,0}\,, \tag{3.17}$$

where $\alpha > \beta > 0$ for all values of M_i. Defining $u = e^{-(\alpha/2)t}$, one finds that

$$ds^2_{n+1} \to -du^2 + u^{2\beta/\alpha} d\Sigma^2_{n,0}\,, \qquad u \to +\infty\,. \tag{3.18}$$

Therefore, $\Sigma_{n,0}$ expands and the curvatures become smaller as $t \to -\infty$. This shows that there is no curvature singularity associated with overall transverse directions.

For $\sigma = 1$, $t \to -\infty$ limit is the same with the $t \to +\infty$ limit; and for all three cases ($\sigma = 0, \pm 1$), $t \to +\infty$ asymptote is the same. In this limit, the metric along overall transverse space takes the form

$$ds^2_{n+1} \to -e^{-\gamma t} dt^2 + e^{\delta t} d\Sigma^2_{n,\sigma}\,, \tag{3.19}$$

where $\gamma > |\delta| \geq 0$, and δ can be positive, negative or zero depending on the constants M_i. For single SM2 and SM5-branes, and for $SM2 \perp SM2(0)$ intersection $\delta > 0$. For $SM2 \perp SM2 \perp SM2(0)$ and $SM2 \perp SM5 \perp SM5(1)$ intersections $\delta \leq 0$. For $SM5 \perp SM5(3)$ and $SM2 \perp SM2 \perp SM5(0)$ intersections $\delta < 0$, and for $SM2 \perp SM5(1)$ and $SM2 \perp SM2 \perp SM5 \perp SM5(0)$ intersections δ can be positive, negative or zero depending on the constants M_i. Defining $u = e^{-(\gamma/2)t}$, the metric becomes

$$ds^2_{n+1} \to -du^2 + u^{-2\delta/\gamma} d\Sigma^2_{n,\sigma}\,, \qquad u \to 0\,. \tag{3.20}$$

Thus, for $\delta < 0$, $\Sigma_{n,\sigma}$ collapses and the geometry becomes singular along these directions. Note that for some intersections, the sign of δ depends on the choice of the constants M_i, and depending on this sign, the asymptotic behavior alters. Also, if one smears out one of the spherical or hyperbolic directions using (3.16), then the coefficients of the exponents in (3.19) will change. For instance, δ may switch sign for fixed M_i after smearing.

In all these solutions there are generic singularities associated with common tangent or relative transverse directions. Specifically, the common tangent directions are always multiplied by negative powers of H. So, as $t \to \pm\infty$, the coefficient of the flat metric along these directions vanishes and the geometry becomes singular. Similarly, along relative transverse directions, there are positive and negative powers of H functions multiplying the flat metric. Therefore, depending on the constants M_i, these directions may collapse, expand or stay flat as $t \to \pm\infty$. If they do collapse, this implies existence of generic singularities associated with relative transverse directions.

As a final technical remark, let us note that if one defines the asymptotic region where the radius of the hyperbola or sphere diverges, then for $\delta < 0$, $t \to \infty$ limit does not obey this criteria. Of course, in this case, one may consider $\Sigma_{n,\sigma}$ as an internal space. Still, however, one may wonder if it is possible to extend the solutions over $t = \infty$ region, and try to determine the maximal possible extensions and global properties. It would also be interesting to understand the motion of test particles on these time dependent backgrounds.

[4] In the following, we will ignore numerical coefficients which do not affect the results.

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4. Conclusions

In this paper, we have constructed intersecting S-brane solutions of $D = 11$ supergravity theory. Our main result is that for S-branes there is an analog of the harmonic function rule. It is remarkable that using these rules, one can construct all standard intersecting SM-brane solutions. It is also very peculiar that S-brane construction rules cannot be applied to non-standard intersections and it seems difficult to obtain explicit solutions corresponding to these configurations. By dimensional reduction and applying S and T-duality transformations, one can also obtain all standard intersecting S-brane solutions of type IIA and IIB string theories.

A crucial point which requires more work to understand is the physical interpretations of the constants in the solutions. Especially, it would be interesting to verify that the parameter M_i is the energy of the corresponding S-brane. Also, it is important to understand the relation between t_i and the moments that S-branes exist. A careful definition of near brane and asymptotic regions is very crucial in fixing these constants. It is interesting to recall that some intersections have different asymptotic behavior depending on the constants characterizing the solution.

Unfortunately, as other examples of S-brane solutions, intersecting backgrounds turn out to be singular. As discussed in [5] there are several possibilities for these singularities. Perhaps the most fortunate fate is the one in which they are smoothed out by non-perturbative α' or g_s effects.

The fundamental problem about these solutions is the question of their stability. Since they do not preserve any supersymmetry, one needs different arguments to claim stability. The presence of a singularity is a bad sign. On the other hand, the fact that spacelike backgrounds corresponding to stable supersymmetric configurations (and not the others) can easily be obtained by an analog of the harmonic function rule gives some hope about the stability of these solutions.

A. Derivation of the rules

The bosonic action of the 11-dimensional supergravity can be written as

$$S = \int d^{11}x\sqrt{-g}(R - \frac{1}{2.4!}F^2) + \mathrm{WZ} \tag{A.1}$$

where the WZ-term is proportional to $\int F \wedge F \wedge A$ and will not be important in this paper. The equations of motion are given by

$$R_{AB} = \frac{1}{2.3!}F_{ACDE}F_B{}^{CDE} - \frac{1}{6.4!}g_{AB}F^2\,, \tag{A.2}$$

$$d * F \sim F \wedge F\,. \tag{A.3}$$

We are interested in configurations where $F \wedge F = 0$ and thus the four-form field equations reduce to $dF = d * F = 0$.

We consider a metric of the following form

$$ds^2 = -e^{2A}dt^2 + \sum_k e^{2C_k} ds_k^2 + e^{2D} d\Sigma_{n,\sigma}^2 \tag{A.4}$$

where ds_k^2 (for $k = 1, 2\ldots$) is the metric on the d_k dimensional flat space. $d\Sigma_{n,\sigma}^2$ is the metric of the n-dimensional unit sphere $\sigma = 1$, unit hyperbola $\sigma = -1$ or flat space $\sigma = 0$. The metric functions are assumed to depend only on t. With respect to the the orthonormal frame $E^0 = e^A dt$, $E^{\alpha_k} = e^{C_k} dy^{\alpha_k}$ and $E^\theta = e^D e^\theta$, where e^θ is an orthonormal frame on $\Sigma_{n,\sigma}$, the Ricci tensor can be calculated as

$$\begin{aligned}
R_{00} &= e^{-2A}\left[-\sum_k d_k(C_k'' + C_k'^2 - C_k'A') - n(D'' + D'^2 - D'A')\right] \\
R_{\alpha_i\beta_i} &= e^{-2A}\left[C_i'' + d_iC_i'^2 - C_i'A' + \sum_{k\neq i} d_kC_k'C_i' + nD'C_i'\right]\delta_{\alpha_i\beta_i} \qquad i = 1, 2\ldots \\
R_{\theta_1\theta_2} &= e^{-2A}\left[D'' + nD'^2 - D'A' + \sum_k d_kC_k'D'\right]\delta_{\theta_1\theta_2} + \sigma(n-1)e^{-2D}\delta_{\theta_1\theta_2}
\end{aligned} \tag{A.5}$$

where $'$ denotes differentiation with respect to t. One can simplify Ricci tensor by fixing t-reparametrization invariance so that

$$A = \sum_k d_kC_k + nD. \tag{A.6}$$

With this gauge choice, the Ricci tensor becomes

$$\begin{aligned}
R_{00} &= e^{-2A}\left[-A'' + A'^2 - \sum_k d_kC_k'^2 - nD'^2\right] \\
R_{\alpha_i\beta_i} &= e^{-2A}\left[C_i''\right]\delta_{\alpha_i\beta_i} \qquad i = 1, 2\ldots \\
R_{\theta_1\theta_2} &= e^{-2A}\left[D''\right]\delta_{\theta_1\theta_2} + \sigma(n-1)e^{-2D}\delta_{\theta_1\theta_2}\,,
\end{aligned} \tag{A.7}$$

and the curvature scalar is

$$R = e^{-2A}\left[2A'' - A'^2 + \sum_k d_kC_k'^2 + nD'^2\right] + \sigma n(n-1)e^{-2D}. \tag{A.8}$$

Right hand side of (A.2) suggests the introduction of functions f_i for each brane which is

$$f_i = -A + \sum d_kC_k + nD\,, \tag{A.9}$$

where the summation is over transverse directions to that brane. The σ term in (A.5) implies the definition of a function g as

$$(n-1)g = A - D\,. \tag{A.10}$$

After these definitions, it is possible to diagonalize the spatial components of Einstein equations by parametrizing the metric functions in terms of f_i and g so that

$$f_i'' = q_i^2 e^{-2f_i}\,, \tag{A.11}$$

$$g'' = -\sigma(n-1)e^{2(n-1)g}\,. \tag{A.12}$$

This parametrization implies (2.4), (2.5) and (2.6) where $H_i = e^{2f_i}$ and $G_{n,\sigma} = e^{-2(n-1)g}$. It is easy to see that (A.11) and (A.12) are integrable and give (2.1) and (2.2). The form of the integration constants in (2.1) and (2.2) together with the relation (2.3) are fixed using the (00) component of the Einstein equations (A.2).

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